

السؤال الأول: (18 درجة)

1- عزم العطالة الكتلي المرجع على الحد القائد:

$$J(\varphi_2) = J_2 \dot{\varphi}_2^2 + J_3 \dot{\varphi}_3^2 + J_6 \dot{\varphi}_6^2 + m_{34} r^2 \dot{\varphi}_3^2 + (m_{45} + m_5) \dot{x}_5^2$$

$$\dot{\varphi}_2 = \frac{d\varphi_2}{dt} = 1, \quad r_2 \varphi_2 = r_3 \varphi_3 \Rightarrow \varphi_3 = \frac{r_2}{r_3} \varphi_2 \Rightarrow \dot{\varphi}_3 = \frac{r_2}{r_3} = \frac{0.15}{0.6} = \frac{1}{4} = 0.25$$

$$r_3 \varphi_3 = -r_6 \varphi_6 \Rightarrow \varphi_6 = -\frac{r_3}{r_6} \varphi_3 = -\frac{r_3 r_2}{r_6 r_3} \varphi_2 \Rightarrow \dot{\varphi}_6 = -\frac{r_2}{r_6} = -\frac{0.15}{0.3} = -\frac{1}{2}$$

$$X_5 \cong L + r \cdot \cos \varphi_3 = L + r \cdot \cos \frac{r_2}{r_3} \varphi_2 \Rightarrow \dot{X}_5 = -r \cdot \frac{r_2}{r_3} \sin \frac{r_2}{r_3} \varphi_2 \Rightarrow \dot{X}_5 = -0.3(0.25) \sin \frac{\varphi_2}{4}$$

$$\dot{X}_5 = -0.075 \sin \frac{\varphi_2}{4} \quad (6)$$

$$m_{34} + m_{45} = m_4 = 8, \quad m_{34} l = m_{45} (L - l) \Rightarrow m_{34} (0.3) = m_{45} (1.2 - 0.3) \Rightarrow m_{34} = 3m_{45}$$

$$m_{45} = 2 \text{ kg}, \quad m_{34} = 6 \text{ kg} \quad (4)$$

$$J(\varphi_2) = 0.12 + 0.6(0.25)^2 + 0.4 \left(-\frac{1}{2}\right)^2 + 6(0.3)^2 (0.25)^2 + (2 + 10) \left(-0.075 \sin \frac{\varphi_2}{4}\right)^2$$

$$J(\varphi_2) = 0.29125 + 0.0675 \sin^2 \left(\frac{\varphi_2}{4}\right)$$

$$J(\varphi_2) = 0.29125 + 0.0675 \left(\frac{1 - \cos\left(\frac{\varphi_2}{2}\right)}{2}\right) \Rightarrow J(\varphi_2) = 0.325 - 0.03375 \cos\left(\frac{\varphi_2}{2}\right) \quad (6)$$

$$J(\varphi_2) = J_m \mp \Delta J \Rightarrow \delta = \frac{\Delta J}{J_m} = \frac{0.03375}{0.325} = 0.10385 \quad (2)$$

السؤال الثاني: (22 درجة)

1- حساب عزم مقاومة العمل المرجع على الحد القائد:

$$Q = \sum_{i=2}^n [F_{Xi} \cdot \dot{X}_i + F_{Yi} \cdot \dot{Y}_i + M_i \cdot \dot{\varphi}_i]$$

$$M_5^{(5)} = \begin{cases} 0 & 0 \leq \varphi_5 \leq \pi \\ -2700 [N \cdot m] & \pi \leq \varphi_5 \leq \frac{3\pi}{2} \\ -1350 [N \cdot m] & \frac{3\pi}{2} \leq \varphi_5 \leq 2\pi \end{cases}$$

دورة العمل (عدد دورات الحد القائد التي توافق دورة واحدة للحد التنفيذي) تساوي $\varphi_5 = 0.5 \cdot 2\pi i = 2\pi \frac{1}{0.5} = 4\pi$

$$M_5^{(2)} = M_5^{(5)} \cdot \dot{\varphi}_5 = M_5^{(5)} \cdot (0.5) = \begin{cases} 0 & 0 \leq \varphi_2 \leq 2\pi \\ -1350 [N \cdot m] & 2\pi \leq \varphi_2 \leq 3\pi \\ -675 [N \cdot m] & 3\pi \leq \varphi_2 \leq 4\pi \end{cases} \quad (6)$$

2- حساب عزم المحرك الضروري في حالة التشغيل المستقر، من مبدأ توازن العمل:

عمل القوى المحركة - عمل القوى المقاومة

عمل كاملة (4π)

$$W_W = -W_M$$

$$W_M = \int_0^{4\pi} M_M d\varphi_2 = 4\pi M_M \quad [N \cdot m]$$

$$W_W = \int_0^{4\pi} M_W d\varphi_2 = \int_0^{2\pi} 0 d\varphi_2 + \int_{2\pi}^{3\pi} (-1350) d\varphi_2 + \int_{3\pi}^{4\pi} (-675) d\varphi_2$$

$$W_W = 0 - 1350(\pi) - 675(\pi) = -2025\pi \quad [N \cdot m] \quad (5)$$

$$4\pi M_M = 2025\pi \Rightarrow M_M = 506.25 \quad [N \cdot m]$$

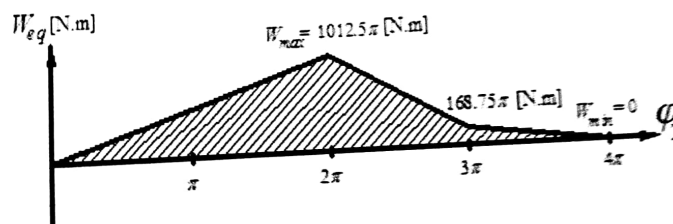
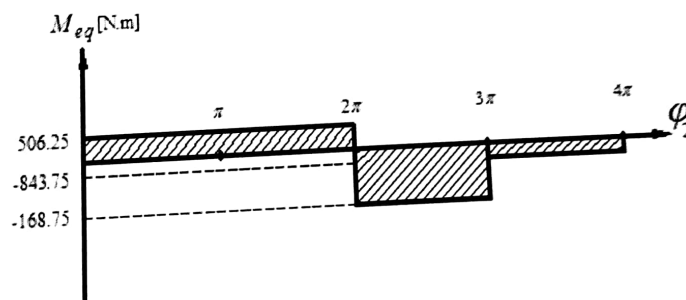
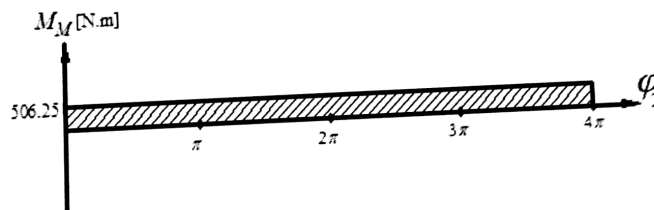
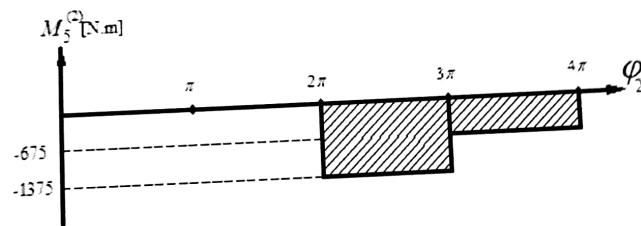
3- العزم المحصل والعمل المحصل:

$$M_{eq} = \begin{cases} 506.25 \quad [N \cdot m] & 0 \leq \varphi_2 \leq 2\pi \\ -843.75 \quad [N \cdot m] & 2\pi \leq \varphi_2 \leq 3\pi \\ -168.75 \quad [N \cdot m] & 3\pi \leq \varphi_2 \leq 4\pi \end{cases}$$

$$W_{max} = \int_0^{2\pi} M_{eq} d\varphi_2 = 506.25(2\pi) = 1012.5\pi = 3180.86 \quad [N \cdot m]$$

$$W_{med} = \int_0^{3\pi} M_{eq} d\varphi_2 = 1012.5\pi - 843.75(\pi) = 168.75\pi = 530.144 \quad [N \cdot m]$$

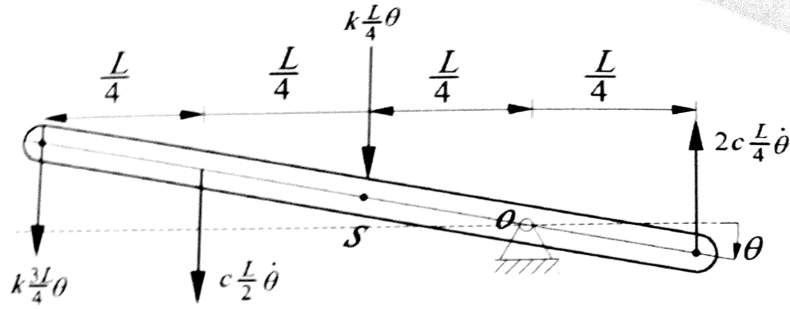
$$W_{min} = 0$$



(4)

(7)

س الثالث: (16 درجة)



(4)

$$J_o = J_s + m \left(\frac{L}{4}\right)^2 = \frac{1}{12} mL^2 + \frac{1}{16} mL^2 = \frac{7}{48} mL^2 \text{ [kg.m}^2\text{]} \quad (2)$$

$$J_o \ddot{\theta} = -2c \frac{L}{4} \dot{\theta} \left(\frac{L}{4}\right) - c \frac{L}{2} \dot{\theta} \left(\frac{L}{2}\right) - k \frac{L}{4} \theta \left(\frac{L}{4}\right) - k \frac{3L}{4} \theta \left(\frac{3L}{4}\right) \Rightarrow$$

$$\frac{7}{48} mL^2 \ddot{\theta} + c \frac{L^2}{8} \dot{\theta} + c \frac{L^2}{4} \dot{\theta} + k \frac{L^2}{16} \theta + k \frac{9L^2}{16} \theta = 0$$

$$\ddot{\theta} + \frac{18c}{7m} \dot{\theta} + \frac{30k}{7m} \theta = 0 \quad (8)$$

$$\omega_n = \sqrt{\frac{30k}{7m}} \left[\frac{\text{rad}}{\text{s}} \right] \quad (2)$$

السؤال الرابع: (24 درجة)

$$m\ddot{x} = -c\dot{x} - kx + F_0 \sin \omega t \Rightarrow \ddot{x} + \frac{c}{m} \dot{x} + \frac{k}{m} x = \frac{F_0}{m} \sin \omega t \quad (3)$$

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(1500)}{60} = 157.08 \frac{\text{rad}}{\text{s}} \quad (2)$$

$$mg = k \Delta \Rightarrow \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{9.81}{0.002}} = 70.036 \frac{\text{rad}}{\text{s}} \quad (3)$$

$$\eta = \frac{\omega}{\omega_n} = \frac{157.08}{70.036} = 2.243, \quad \zeta = 0.2$$

$$TR = \frac{F_T}{F_0} = \frac{1 + (2\zeta \frac{\omega}{\omega_n})^2}{\sqrt{(1 - (\frac{\omega}{\omega_n})^2)^2 + (2\zeta \frac{\omega}{\omega_n})^2}} = \frac{1 + (2 \times 0.2 \times 2.243)^2}{\sqrt{[1 - (2.243)^2]^2 + (2 \times 0.2 \times 2.243)^2}} = 0.3253 \quad (6)$$

$$TR = \frac{F_T}{F_0} = \frac{F_T}{2450} = 0.3253 \Rightarrow F_T = 796.985 \text{ N} \quad (2)$$

$$x = X \sin(\omega t - \phi) \Rightarrow mg = k \Delta \Rightarrow k = \frac{mg}{\Delta} = \frac{1000 \times 9.81}{0.002} = 4905000 \frac{\text{N}}{\text{m}} \quad (2)$$

$$X = \frac{\frac{F_0}{k}}{\sqrt{(1 - (\frac{\omega}{\omega_n})^2)^2 + (2\zeta \frac{\omega}{\omega_n})^2}} = \frac{\frac{2450}{4905000}}{\sqrt{(1 - (2.243)^2)^2 + (2 \times 0.2 \times 2.243)^2}} \Rightarrow X = 1.2095 \times 10^{-4} \text{ m} \quad (6)$$