سلم تصحيح مقرر الرياضيات ٤
سنة ثانية إنتاج وتصميم

السؤال الأول (20 درجة)

$$f(x)=\sum_{n=-\infty}^{+\infty} C_n \cdot e^{inx} \quad (5$$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) \cdot e^{-inx}\, dx \; ; \; n=0,\pm1,\pm2,\dots \quad (5$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} 0 \cdot e^{-inx}\,dx + \frac{1}{2\pi}\int_{0}^{\pi} 1 \cdot e^{-inx}\,dx$$

$$= 0 + \frac{1}{2\pi}\int_{0}^{\pi} e^{-inx}\,dx = \frac{1}{2\pi}\left[\frac{e^{-inx}}{-in}\right]_0^{\pi}$$

$$= \frac{1}{2\pi}\left[\frac{e^{-in\pi}}{-in} - \frac{e^{0}}{-in}\right] = -\frac{1}{2\pi i n}\left[e^{-in\pi}-1\right]$$

$$= -\frac{1}{2\pi i n}\left[\cos(n\pi) - i\sin(n\pi) - 1\right] \quad (5$$

$$= -\frac{1}{2\pi i n}\left[(-1)^n - 1\right] = \frac{1}{2\pi i}\,\frac{1-(-1)^n}{n}$$

ومنه فسلسلة فورييه العقدية لهذه الدالة هي:

$$f(x) = \frac{1}{2\pi i}\sum_{n=-\infty}^{+\infty} \frac{1-(-1)^n}{n}\, e^{inx} \quad (5$$

السؤال الثاني (20 درجة)

الخطأ بالثابت تأخذ 3 فقط

$$F_c(\alpha) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\cos\alpha x\, dx \quad (5$$

$$= \sqrt{\frac{2}{\pi}}\int_0^{\infty} e^{-x}\cos\alpha x\, dx$$

$u = e^{-x} \Rightarrow du = -e^{-x}\,dx$

$dv = \cos\alpha x \Rightarrow v = \frac{1}{\alpha}\sin\alpha x \quad \Big\} \Rightarrow$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{e^{-x}}{\alpha}\sin\alpha x\Big|_0^{\infty} + \frac{1}{\alpha}\int e^{-x}\sin\alpha x\, dx\right]$$

$$= \sqrt{\frac{2}{\pi}}\left[0 + 0 + \frac{1}{\alpha}\underbrace{\int e^{-x}\sin\alpha x\, dx}_{y}\right]$$

$u = e^{-x} \Rightarrow du = -e^{-x}\,dx$

$dv = \sin\alpha x \Rightarrow v = -\frac{1}{\alpha}\cos\alpha x$

$$\Rightarrow = \sqrt{\frac{2}{\pi}}\left[\frac{1}{\alpha}\left(-\frac{e^{-x}}{\alpha}\cos\alpha x\Big|_0^{\infty} - \frac{1}{\alpha}\int_0^{\infty} e^{-x}\cos\alpha x\right)\right]$$

$$\Rightarrow I = \frac{1}{\alpha}\left(0 + \frac{1}{\alpha} - \frac{1}{\alpha} I\right)$$

$$\Rightarrow I = \frac{1}{\alpha^2} - \frac{1}{\alpha^2} I \Rightarrow \left(1 + \frac{1}{\alpha^2}\right) I = \frac{1}{\alpha^2}$$

$$\Rightarrow \frac{\alpha^2+1}{\alpha^2} I = \frac{1}{\alpha^2} \quad (5$$

$$\Rightarrow I = \frac{1}{\alpha^2+1} \Rightarrow F = \sqrt{\frac{2}{\pi}} \cdot \frac{1}{\alpha^2+1}$$

$$F^{-1} = \sqrt{\frac{2}{\pi}}\int_0^{\infty} F \cdot \cos\alpha x\, d\alpha \quad (5$$

$$e^{-x} = \frac{2}{\pi}\int_0^{\infty} \frac{\cos\alpha x}{\alpha^2+1}\, d\alpha$$

$$\Rightarrow \int_0^{\infty} \frac{\cos\alpha x}{\alpha^2+1}\, d\alpha = \frac{\pi e^{-x}}{2} \quad (5$$

السؤال الثالث (20 درجة) لدينا:

$$\beta(m,n) = \int_0^1 t^{m-1}(1-t)^{n-1}\, dt$$

لنفرض $t = \frac{y}{1+y}$ (5

$$dt = \frac{(1+y)\,dy - y\,dy}{(1+y)^2} = \frac{dy}{(1+y)^2}$$

$$1-t = \frac{1}{1+y}$$

$y = \frac{t}{1-t} \Leftarrow t = \frac{y}{1+y}$ وبما

$y = 0 \Leftarrow t = 0$

$y \to \infty \Leftarrow t = 1$ (5

$$\beta(m,n) = \int_0^{\infty}\left(\frac{y}{1+y}\right)^{m-1}\left(\frac{1}{1+y}\right)^{n-1}\frac{dy}{(1+y)^2}$$

$$= \int_0^{\infty}\frac{y^{m-1}}{(1+y)^{(m-1)+(m-1)+2}}\, dy \quad (5$$

(5

$$= \int_0^{\infty}\frac{y^{m-1}}{(1+y)^{m+n}}\, dy$$

و.هـ.م

السؤال الرابع (5 درجات)

باستخدام المبرهنة الثانية

$$I = \int_0^\infty \mathcal{L}\left[e^{-at} - e^{-bt}\right] ds \quad (5)$$

$$= \int_0^\infty \left[\frac{1}{s+a} - \frac{1}{s+b}\right] ds \quad (5)$$

$$= \left[\ln|s+a| - \ln|s+b|\right]_0^\infty \quad (5)$$

$$= \left[\ln\left|\frac{s+a}{s+b}\right|\right]_0^\infty = \ln\left|\frac{a}{b}\right| - \ln 1 \quad (5)$$

$$= \ln\left|\frac{b}{a}\right| = -\ln\left|\frac{a}{b}\right|$$

السؤال الخامس (5 درجات)

$$f(t) = t^{-\frac{1}{2}}\left[\frac{e^{\omega t} - e^{-\omega t}}{2}\right] \quad (5)$$

$$\mathcal{L}[f(t)] = \mathcal{L}\left[\frac{1}{2}t^{-\frac{1}{2}}\left(e^{\omega t} - e^{-\omega t}\right)\right]$$

$$= \frac{1}{2}\left[\mathcal{L}\left[t^{-\frac{1}{2}}e^{\omega t}\right] - \mathcal{L}\left[t^{-\frac{1}{2}}e^{-\omega t}\right]\right] \quad (5)$$

$$= \frac{1}{2}\frac{\Gamma(\frac{1}{2})}{(s-\omega)^{\frac{1}{2}}} - \frac{1}{2}\frac{\Gamma(\frac{1}{2})}{(s+\omega)^{\frac{1}{2}}} \quad (5)$$

$$= \frac{\sqrt{\pi}}{2(s-\omega)^{\frac{1}{2}}} - \frac{\sqrt{\pi}}{2(s+\omega)^{\frac{1}{2}}}$$

مدرسة المقرر

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