سلم تصحيح مقرر{الرياضيات(٣)} ثانية:<معادن> الفصل الثاني لعام/ ٢٠٢٣ / م

ج١): (٦٠): (١-١):

$$\xrightarrow{\div(-x^2y^3)} \frac{y'}{y^3}-\frac{1}{x}\cdot\frac{1}{y^2}=\frac{-\sin x}{x^2}$$

(خطية):

$$\underset{z'=\frac{-2y'}{y^3}}{\overset{z=\frac{1}{y^{n-1}}=\frac{1}{y^2}}{\longrightarrow}} z'+\frac{2}{x}\cdot z=\frac{2\sin x}{x^2} \Rightarrow ze^{2\int\frac{dx}{x}}=zx^2=$$

$$=2\int e^{2\int\frac{dx}{x}}\frac{\sin x}{x^2}dx \to zx^2=2\int \sin x\,dx=-\cos x+c \Rightarrow y=\frac{x}{\sqrt{-2\cos x+c}}$$

(٢-١):

$$(D^2+4D+5)y=e^{-2x}\sin x \to k^2+4k+5=0,(k+2)^2=-1=i^2,k=-2\pm i,y_c=e^{-2x}(A\cos x+B\sin x).$$

$$y_p=\mathrm{Im}\left[\frac{1}{(D^2+4D+5)}\right]e^{(-2+i)x}=\mathrm{Im}\left[\frac{1}{[D-(-2+i)]\underbrace{[D-(-2-i)]}_{\Phi(\lambda)\neq 0}}\right]e^{(-2+i)x}=\mathrm{Im}\left[\frac{-i}{2}\frac{xe^{-2x}}{1!}(\cos x+i\sin x)\right]=\frac{-xe^{-2x}\cos x}{2}$$

$$y=y_c+y_p=e^{-2x}\left[(A\cos x+B\sin x)+\frac{-x\cos x}{2}\right]$$

(٣-١):

$$V=e^x\sin y,V_x=e^x\sin y,V_{xx}=e^x\sin y,V_y=e^x\cos y,V_{yy}=-e^x\sin y \Rightarrow V_{xx}+V_{yy}=0$$

إذاً V توافقية ولنوجد مرافقتها التوافقية U بالشكل:

$$\frac{\partial V}{\partial y}=\frac{\partial U}{\partial x}=e^x\cos y \xrightarrow{\int} U=e^x\cos y+\varphi(y),\frac{\partial U}{\partial y}=\frac{-\partial V}{\partial x}\to -e^x\sin y+\varphi'(y)=-e^x\sin y \to$$

$$\varphi'(y)=0,\varphi(y)=c \to U=e^x\cos y+c \Rightarrow f(z)=U+iV=e^x(\cos y+i\sin y)+c \underset{y\to 0}{\overset{x\to z}{=}} e^z+c$$

ج٢): (٤٠): (١-٢):

$$I=\int_{-\infty}^{\infty}\frac{\sin\alpha x}{x(x^2+a^2)}dx;[\alpha>0,a>0]\to f(z)e^{i\lambda z}=\frac{e^{iaz}}{z(z^2+a^2)};z_1=0(on\,0X),z_2=ia(up\,0X),z_3=-ia(down\,0X)$$

$$\mathrm{Res}\left[f(z)e^{iaz};z_1=0\right]=\lim_{z\to 0}\frac{ze^{iaz}}{z(z^2+a^2)}=\frac{1}{a^2},\mathrm{Res}\left[f(z)e^{iaz};z_2=ia\right]=\lim_{z\to ia}\frac{(z-ia)e^{iaz}}{z(z-ia)(z+ia)}=\frac{-1}{2a^2e^{aa}}\Rightarrow$$

$$I=\mathrm{Im}\left\{2\pi i\,\mathrm{Res}\left[f(z)e^{iaz};z_2=ia\right]+\pi i\,\mathrm{Res}\left[f(z)e^{iaz};z_1=0\right]\right\}=\mathrm{Im}\left\{2\pi i\left(\frac{-1}{2a^2e^{aa}}\right)+\pi i\left(\frac{1}{a^2}\right)\right\}=\pi\left(\frac{-1}{a^2e^{aa}}+\frac{1}{a^2}\right)$$

(٢-٢):

$$J=\int_0^{2\pi}\frac{d\theta}{\sin\theta+3}\underset{\sin\theta=z^2-1/2iz}{\overset{z=e^{i\theta},d\theta=dz/iz}{=}}2\oint_{|z|=1}\frac{dz}{z^2+6iz-1},z_{1,2}=\left(-3\mp 2\sqrt{2}\right)i;z_1=\left(-3+2\sqrt{2}\right)i\in|z|=1,$$

$$\mathrm{Res}(f,z_1)=\frac{1}{2z_1+6i}=\frac{1}{4i\sqrt{2}}\Rightarrow I=2\pi i(2)\left(\frac{1}{4i\sqrt{2}}\right)=\frac{\pi}{\sqrt{2}}$$

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