تحديد سماكة المادة المتحولة الطور خلال عملية التجمد حول اسطوانة دورانية تحليلياً وعددياً وتجريبياً

م. علاء عواد*   أ.د.م. نصر ياسين**

الملخص

إن نظام تخزين الطاقة الحرارية الباردة هو وسيلة مبتكرة لتخزين الطاقة ليلاً خارج اوقات الذروة لاستخدامها في اوقات الذروة نهاراً في العديد من المواقع. ومن المعلوم أن تكييف وتبريد الهواء يشكل الاستهلاك الأعظمي للطاقة الكهربائية في فصل الصيف حيث في بعض المناطق تمثل ما يصل إلى نصف معدل الاستهلاك الطاقي خلال ساعات الذروة في منتصف اليوم، وبما أن المرافق لديها ليلاً قدرة توليد كهربائية، فإن الكهرباء المولدة خلال خارج اوقات الذروة تكون أقل تكلفة بكثير. الهدف الرئيس من هذا البحث هو إيجاد علاقة تربط بين سماكة الطور الصلب المتشكل (الجليد) عند استخدام الماء كمادة متغيرة الطور مع الزمن. تم استخدام طريقة المقاومات الحرارية لإيجاد الحل التحليلي. تم استخدام برنامج ANSYS15 لحل المعادلات الناظمة للمسألة المدروسة بطريقة الحجوم المنتهية، وكانت النتائج بين الطريقتين السابقتين متقاربة بنسبة خطأ 9.5% في سماكة الطور الصلب المتشكل بعد 8 ساعات عمل. لمعرفة مدى تطابق الحلول النظرية مع الواقع، تم إجراء تجربة عملية باستخدام محلول غليكول إيثيلين كوسيط تبريد ثانوي (سائل ناقل الحرارة) وكان الخطأ النسبي في سماكة الطور الصلب المتشكل مع الزمن بعد 8 ساعات عمل بين الطريقتين التحليلية والتجريبية 13.5%, وبين الطريقتين الحاسوبية العددية والتجريبية 2.7%.

* طالب دكتوراه في قسم الميكانيك العام في كلية الهندسة الميكانيكية والكهربائية – جامعة دمشق.

** أستاذ في قسم الميكانيك العام في كلية الهندسة الميكانيكية والكهربائية – جامعة دمشق.

Estimate the Solidification Phase Thickness Around a Circular Cylinder Analytically, Numerically and Experimentally

Eng. Alaa Awad* **Dr. Nasr Yasin****

Abstract

CTES (Cold Thermal Energy Storage) is an innovative way of storing night-time off-peak energy for daytime peak use. In many locations, demand for electrical power peaks during summer. Air-conditioning and refrigerating is the main reason, in some areas accounting for as much as half of the power demand during the hot mid-day hours when electricity is most expensive. Since, at night, utilities have spare electrical generating capacity, electricity generated during this "off-peak" is much less expensive. The main objective of this research is to find a relationship between the thickness of the solid phase formed (ice) when using water as a phase change material with time. Thermal resistances method was used to find the analytical solution. ANSYS15 program was used to solve the equations of the problem by the finite volumes method, and the results between the two previous methods were close with an error of 9.5% in the thickness of the solid phase formed after 8 working hours. To find out the accuracy of the theoretical solutions, an experiment was done using an ethylene glycol solution as a secondary cooling medium (heat transfer fluid). The error in the thickness of the solid phase formed with time after 8 working hours between the analytical and experimental methods was 13.5%, and between the numerical and experimental methods was 2.7%.

Keywords: Thermal Ice Storage System (TISS), Thermal resistance, Phase Change Material (PCM).

* PHD student in Mechanical Department at Mechanical and Electrical Engineering Faculty - Damascus University

** Professor in Mechanical Department at the Mechanical and Electrical Engineering Faculty - Damascus University

1. Introduction

The world energy consumption is increasing rapidly, and the developed countries have become heavily reliant upon energy in all activities. It is well accepted now that electrical energy is the most versatile form of energy and its conservation will solve part of the world energy problems. Moreover, in most cases the “offer” and “demand” in thermal energy do not coincide in time. Hence, the obvious key to solve both problems is an efficient storage for low temperature thermal energy.

Bareiss et al. [1] studied melting enhancement above a heated horizontal cylinder by means of numerical and perturbation methods. Zhang et al. [2] studied analytically and experimentally melting of an unfixed solid PCM inside a horizontal tube. Murray et al. [3] developed a successful method for the numerical solution of freezing adjacent to a flat plate. Habeebullah [4] conducted an experimental study on ice formation around a horizontal long copper tube (L = 12.3 m with three returned bends of 180° and d = 19.5 mm) immersed in water, he found that there was a slope of the ice thickness, in which the axial distance depended on time but varied with coolant flow rate and Stanton and Biot numbers. The axial growth rate of ice was distinct for low values of the coolant Reynolds number and short freezing times. He also discovered an unexpected enlargement of ice thickness on the surface of the tube bends. Sait et al. [5,6] performed an experimental investigation on the freezing of water falling film on a vertical bank of horizontal cold tubes. The brine flows into the concentric tubes in parallel. The authors focused their work on ice formation characteristics and heat transfer for the three main modes of falling film: droplets, jets and sheets. These researchers determined that the formation of ice depends on falling film and coolant flow rates. In addition to that, he found that the thermal resistance of ice controlled the overall heat transfer coefficient. Hosseini et al. [7] experimentally and numerically investigated the melting process of the commercial paraffin RT50 inside a shell-and-tube heat exchanger with water flowing through the tube. It was found that natural convection inside the liquid PCM could enhance the heat storage process.

Yingxin et al. [8] developed a theoretical model to analyze the heat transfer during the melting process. A series of experiments on internal melting of unrestrained ice around a fixed horizontal tube were reported. The temporal geometric shape, melting rates under various experimental conditions were determined using a photograph technique. Sait et al. [9] investigated ice formation on cold vertical banks of horizontal tubes subjected to falling-film – jet mode– experimentally. In the charging process, a set of internally cooled vertical banks of horizontal tubes of brine is subjected to a falling film of water. The formed ice is periodically observed, photographed and measured in falling-film jet mode at specific internal coolant (ethylene–glycol solution) flow rates and temperatures. In the discharge process, the same solution is heated and used internally to release ice. Different thicknesses of the released ice were observed and measured. The maximum quantity of released ice is obtained and the optimum ice formation is determined. Tien et al. [10] were concerned with the transient location of a freezing front in the region outside an infinitely long circular cylinder. The surroundings were assumed to be initially at a uniform temperature above the freezing temperature T_f. The surface of the cylinder was suddenly reduced to, and/or maintained at, a constant temperature, below T_∞. The frozen and unfrozen regions were assumed to have constant, but different, heat capacities and thermal conductivities. The density is assumed to be the same in both phases. Heat transfer is by conduction only, and the latent heat is all absorbed at T_f .

M. Yari et al. [11] used a numerical method for solving energy equation and describing solidification phenomenon around a circular pipe. Further the effect of pipe surface temperature and initial water temperature to increase freezing zone have been analyzed. Ismail et al. [12] studied a numerical study validated by experimental measurements on the solidification of PCM along a horizontal tube by using the boundary immobilization technique. Jiajia Liu et al. [13] performed a transient simulation of the charging processes based on a two-dimensional numerical model, and the melting processes of the storage units with staggered tube bundle structure and parallel tube bundle structure are compared with that of flat plate structure. Sugawara et al. [14] investigated Freezing/melting of water/ice around a horizontal cylinder placed in a square cavity of the inner side length numerically.

in this research, we will present the relation between solidification phase thickness around a circular cylinder (δ) and time analytically, numerically and experimentally.

2. Analytical Solution

2-1. Mathematical Model.

Consider a circular cylinder of external radius (r_e) surrounded by PCM of a temperature equal to the phase change temperature $\left(T_f\right)$ and heat transfer fluid (HTF) flow inside circular cylinder the temperature of (HTF) lower than the PCM temperature $\left(T_\infty < T_f\right)$, as seen in Figure (1).

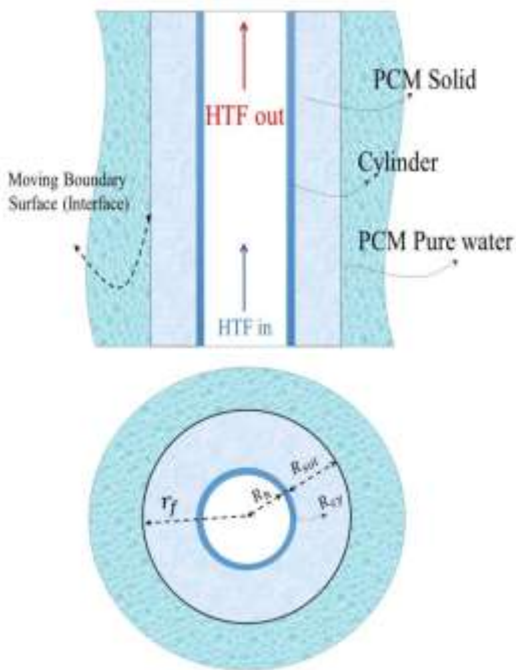

Fig. (1) Schematic view of a cylindrical thermal resistance network.

The heat exchange between the HTF and PCM is dominated by convection heat transfer. The PCM starts to solidify from the outer boundary of the container and the phase change front moves towards the surface of the tank. Three assumptions are taken into considerations:

1- The temperature of the free-stream of HTF (T_∞) and the heat transfer coefficient (h) are constants.
2- The heat transfer process inside the container is conduction dominant and in the radial direction.
3- The densities of solid and liquid phases of PCM are equal.

The thermal resistant network has three resistances to deliver the heat from the PCM to HTF, as per Fig (1):

1. Convection resistance at the surface of the cylinder's container (R_h).
2. Conductive resistance at the shell walls (R_{cy}).
3. Conductive resistance for the PCM solid phase (R_{sol}).

The heat delivered from the cylinder to the HTF (solidification) can be calculated by the thermal resistance network as follow:

$$Q_f = \frac{T_f - T_\infty}{R_{cy} + R_{sol} + R_h} \tag{1}$$

$$R_{cy} = \frac{\ln(r_{out}/r_{in})}{2\pi l k_{cy}} \tag{2}$$

$$R_{sol} = \frac{\ln(r_f/r_{out})}{2\pi l k_{sol}} \tag{3}$$

$$R_h = \frac{1}{2\pi r_{in} l h} \tag{4}$$

Also the energy balance around the cylinder is:

$$Q_f = \rho L \frac{dv}{dt} = 2\pi\rho l L r_f \frac{dr_f}{dt} \tag{5}$$

Substituting equation (1) into (5), we get:

$$2\pi\rho l L r_f \frac{dr_f}{dt} = \frac{T_f - T_\infty}{\frac{\ln(r_{out}/r_{in})}{2\pi l k_{cy}} + \frac{\ln(r_f/r_{out})}{2\pi l k_{sol}} + \frac{1}{2\pi r_{in} l h}} \tag{6}$$

By using first order separation of variables for a homogeneous linear differential equation, the solution would be:

$$\frac{1}{2}\rho L r_f^2 \left[k_{cy}k_{sol} + r_{in}hk_{sol}\ln(r_{out}/r_{in}) + r_{in}hk_{cy} * \left(\ln(r_f/r_{out}) - \frac{1}{2}\right)\right] = r_{in}hk_{cy}k_{sol}(T_f - T_\infty)t + C \tag{7}$$

By using the following initial condition in order to find the constant C in equation (7):

$$r_f = r_{out} \quad at \quad t = 0 \tag{8}$$

The equation (7) becomes:

$$t = \frac{\rho L(r_f^2 - {r_{out}}^2)}{2(T_f - T_\infty)}\left(\frac{1}{r_{in}h} + \frac{\ln(r_{out}/r_{in})}{k_{cy}} - \frac{1}{2k_{sol}}\right) + \frac{\rho L r_f^2}{2k_{sol}(T_f - T_\infty)}\ln(r_f/r_{out}) \tag{9}$$

If we assume that

$$C_1 = \frac{\rho L}{2(T_f - T_\infty)}\left(\frac{1}{r_{in}h} + \frac{\ln(r_{out}/r_{in})}{k_{cy}} - \frac{1}{2k_{sol}}\right) \tag{10}$$

$$C_2 = \frac{\rho L}{2k_{sol}(T_f - T_\infty)} \tag{11}$$

The time equation (9) becomes:

$$t = C_1(r_f^2 - {r_{out}}^2) + C_2\ln(r_f/r_{out})r_f^2 \tag{12}$$

2-2. Average convective heat transfer coefficient.

To solve equation (11), we need to obtain the heat transfer coefficient form a flow inside a cylinder, [10] recommends the following relations for the heat transfer coefficient:

if $\mathrm{Re_f} < 2300$

$$\mathrm{Nu} = 3.66 + \frac{(0.19\mathrm{RePr}\, {}^{d}/{}_{l})^{0.8}}{1 + 0.117(\mathrm{RePr}\, {}^{d}/{}_{l})^{0.467}} \qquad (13)$$

if $2300 < \mathrm{Re_f} < 10^{4}, 5 < \mathrm{Pr_f} < 500$

$$\mathit{and}\ 0.05 < \left(\frac{\mathrm{Pr_f}}{\mathrm{Pr_w}}\right) < 20$$

$$\mathrm{Nu} = 0.012\left(\mathrm{Re_f}^{0.87} - 280\right)\mathrm{Pr_f}^{0.4} * \left[1 + \left(\frac{d}{L}\right)^{\frac{2}{3}}\right](\mathrm{Pr_f}/\mathrm{Pr_w})^{0.11} \qquad (14)$$

The fluid properties, in this case, are taken from ref [16] and evaluated at the free-stream temperature (T_∞) .

2-3. Results

We will show the relationship between **Radial Interface Position (Solidification Phase Thickness)** and the time. The thermos-physical properties shown in Table (1) are used in eq. (12).

Figure (2) shows the relation between solidification phase thickness (δ) and time. At ($t = 0$), the solidification starts at the cylinder outer surface ($\delta = 0$). The thermos-physical properties of Ethylene Glycol (EG) which is the heat transfer fluid are taken from ASHRAE [16]. Figure (2) shows a solidification phase thickness after 8 hours equal to 4.2 (cm).

Table (1): Thermo-Physical Properties.

Water Solid Properties

Water initial temperature	$273.15\ (K) = 0\ (°C)$
Thermal conductivity of solid PCM	$1.88\ (W.m^{-1}.K^{-1})$
Density of solid PCM	$1000\ (kg.m^{-3})$
Heat of fusion	$334000\ (J/kg)$
HTF (EG) Properties	
Volume Concentration	40%
Velocity	$1.0\ (m.s^{-1})$
Initial temperature	$261.15\ (K) = -12\ (°C)$
Cylinder Materials	
Radius	$2\ (cm)$
Thermal conductivity	$110\ (W/m.K)$
Wall Thickness	$1\ (mm)$

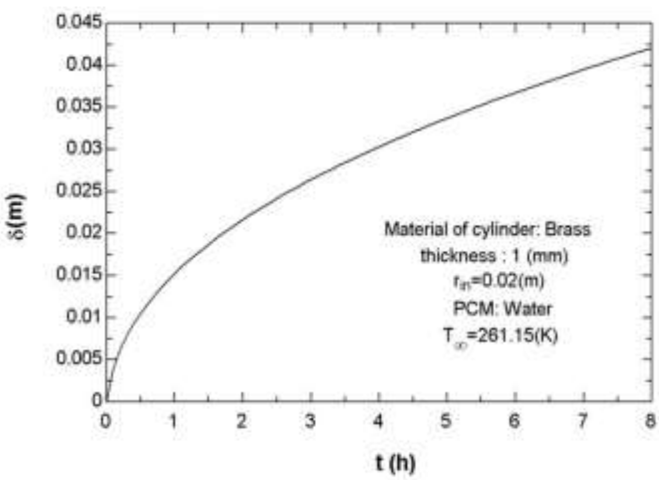

Fig. (2) The relationship between solidification phase thickness and time.

3. Numerical solution

The numerical solution was done by ANSYS program, we used The thermos-physical properties shown in Table (1) Figure (3) shows the relation between solidification phase thickness (δ) and time. At $(t = 0)$, the solidification starts at the cylinder outer surface $(\delta = 0)$. The thermos-physical properties of Ethylene Glycol (EG) which is the heat transfer fluid are taken from ASHRAE [16]. table (2) shows a relation between solidification phase thickness and time every hour. the solidification was after 8 hours equal to 3.8 (cm).

Table (2): Relation between solidification phase thickness and time.

solidification phase thickness $(\boldsymbol{cm})$	**Time** $[\boldsymbol{h}]$
0.7	1
1.4	2
2	3
2.6	4
3	5
3.4	6
3.6	7
3.8	8

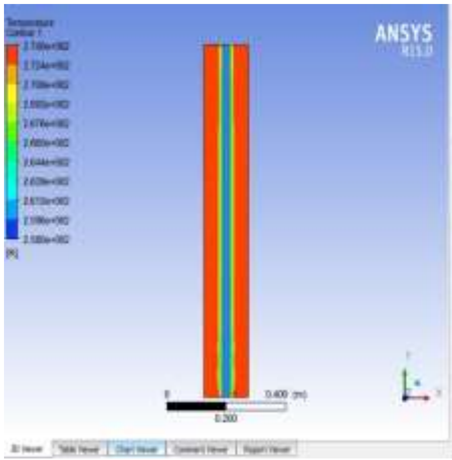

Fig. (4) Temperature distribution by the program after 2 hour.

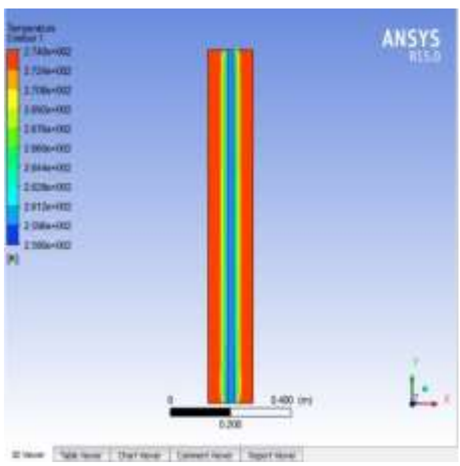

Fig. (5) Temperature distribution by the program after 4 hour.

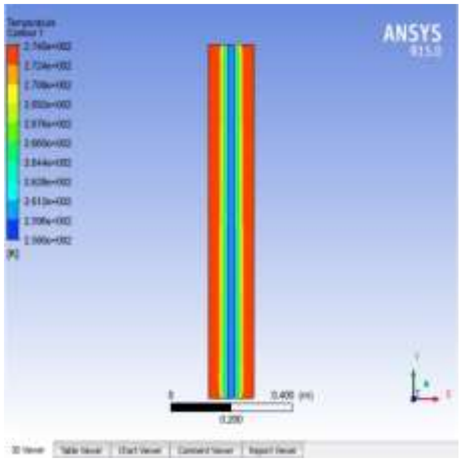

Fig. (6) show the temperature distribution by the program after 6 hour.

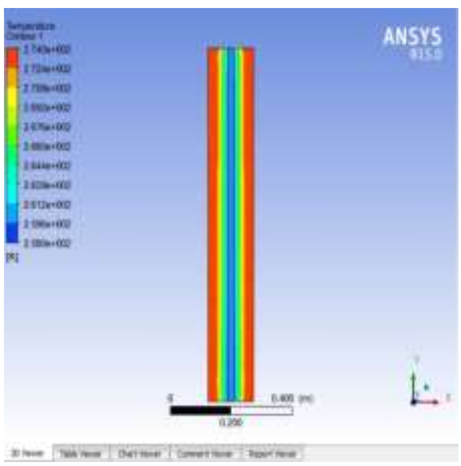

Fig. (7) Temperature distribution by the program after 8 hour.

4. Experiment

The experimental solution was done by used a glass box containing a brass tube of length 1 m. The outside tube diameter was 2.2cm and the inner diameter 2 cm. The glass box was 10 × 10 × 100 cm, fig. (8).

Fig. (8) Experimental water tank.

During the ice formation process, the cooling fluid was a glycol solution of 40%wt concentration whose inlet temperature was −12 °C and a flow rate of 0.5 m3·h−1.

Table (3) shows the relation between solidification phase thickness and time. The solidification phase thickness after 8 hours was equal to 3.7 (cm).

Table (3): Relation between solidification phase thickness and time.

solidification phase thickness (cm)	**Time** $[h]$
0.6	1
1.3	2
1.8	3
2.5	4
2.9	5
3.1	6
3.4	7
3.7	8

5. Conclusion

The main objective of this research is to find a relationship between the thickness of the solid phase formed (ice) when using water as a phase change material with time. Thermal resistances method was used to find the analytical solution. ANSYS15 program was used to solve the equations of the problem by the finite volumes method, and the results between the two previous methods were close with an error of 9.5% in the thickness of the solid phase formed after 8 working hours. To find out the accuracy of the theoretical solutions, an experiment was done using an ethylene glycol solution as a secondary cooling medium (heat transfer fluid). The error in the thickness of the solid phase formed with time after 8 working hours between the analytical and experimental methods was 13.5%, and between the numerical and experimental methods was 2.7%.

Fig. (9) Shows the relation between thickness of ice (meter) and time (hours) for Analytical, Numerical and Experimental Methods.

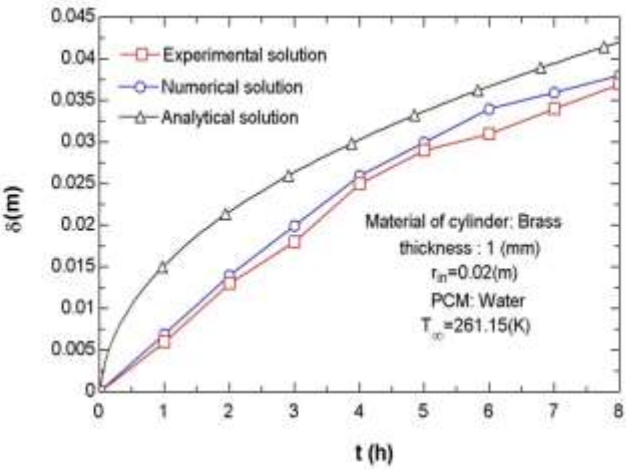

Fig. (9) Relation between thickness of ice and time for Analytical, Numerical and Experimental Methods.

Nomenclature

Q_f heat flux (W)
L latent solidification heat $(J.kg^{-1})$
V volume (m^3)
r_{in} internal radius (m)
r_{out} external radius (m)
r_f interface radius (m)
δ solidification phase thickness (r_f-r_{out}) (m)
R thermal resistance $(K.W^{-1})$
k thermal conductivity $(W.m^{-1}.K^{-1})$
h convective heat transfer coefficient$(W.m^{-2}.K^{-1})$
T temperature (K)
T_f phase change temperature (K)
$T_\propto$ free-stream temperature (K)
t time (s)
l length of cylinder(m)
C_p specific heat$(J.kg^{-1}.K^{-1})$
g gravity $m.s^{-2}$
v_{htf} heat transfer velocity $(m.s^{-1})$
EG ethylene glycol

Greek symbols

ρ density $(kg.m^{-3})$
μ dynamic viscosity$(Pa.s)$
ν kinematic viscosity$(m^2.s^{-1})$
α thermal Diffusivity $(m^2.s^{-1})$

Subscripts

sol solid
h free convection on external radius
cy cylinder

Dimensionless numbers

Nu Nusselt number $Nu = \frac{2hR_e}{k}$
Ra rayleigh number $\mathrm{Ra} = \frac{2\mathrm{VR_e}}{\mathrm{v}}$
Pr Prandtl number $r = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$
Re Reynolds Number $Re = \frac{2\mathrm{V_{HTF}R_e}}{\nu}$

References

[1] BAREISS M, BEER, H, 1984, An analytical solution of the heat transfer process during melting of an unfixed solid phase change material inside a horizontal tube. Int J Heat Mass Transfer, VOL 27. 739–746

[2] ZHANG GP, WEINBAUM, S, 1985, Quasi-three-dimensional steady-state analytic solution for melting or freezing around a buried pipe in a semi-infinite medium. ASME J Heat Transf, VOL. 107. 245–247

[3] MURRAY, W. D, LANDIS, F, J. Heat Transfer.

[4] HABEEBULLAH, BA, 2007, An experimental study on ice formation around horizontal long tubes. Int J Refrigerating, VOL.30. 789–801.

[5] SAIT, HH, HUSSAIN, A, SELIM, AM, 2012, Experimental investigation on freezing of water falling film on vertical bank of horizontal cold tubes. J Thermal Sci Eng Appl, VOL.4. 1-7.

[6] SAIT, HH, 2012, Heat transfer analysis and effects of feeding tubes arrangement, falling film behavior and backsplash on ice formation around horizontal tubes bundles. Energy Convers Manage. VOL.73. 317–328.

[7] HOSSEINI, M.J, 2012, A combined experimental and computational study on the melting behavior of a medium temperature phase change storage material inside shell and tube heat exchanger, Int. Commun. Heat Mass Transf. VOL. 39. 1416–1424.

[8] 10.1016/j.icheatmasstransfer.2012.07.028.

[9] SAIT, H2013, Heat transfer analysis and effects of feeding tubes arrangement , falling film behavior and backsplash on ice formation around horizontal tubes bundles. Energy Conversion and Management, VOL. 73. 317–328.

[10] YINGXIN, H, 2001, Heat transfer processes during an unfixed solid phase change material melting outside a horizontal tube, Int. J. Therm. VOL. 1. 1–14.

[11] SAIT, H, 2014, Charging and discharging characteristics of cool thermal energy storage system with horizontal pipes using water as phase change material.

[12] YARI, M, Monsef , H, 2010, Numerical Analysis Of Water Solidification Around Horizontals Tubes Using For Ice Storage Systems, Journal of Economics and Engineering, ISSN: 2078-0346.

[13] KAMAL A.R. ISMAIL, A.M 2014, Experimentally validated two dimensional numerical model for the solidification of PCM along a horizontal long tube. International Journal of Thermal Sciences, VOL. 75.

[14] JIAJIA, L, 2017. Numerical investigation on the heat transfer enhancement of a latent heat thermal energy storage system with bundled tube structures. Applied Thermal Engineering, VOL. 112. 820–831.

[15] SUGAWARA, Y, Komatsu, H, 2008. Melting and freezing around a horizontal cylinder placed in a square cavity. Heat Mass Transfer, VOL. 45. 83–92

[16] ASHRAE Handbook—Fundamentals (2013). CH31. “PHYSICAL PROPERTIES OF SECONDARY COOLANTS (BRINES)”.